ECS 455: Quiz 3 Solution

Instructions

1. Separate into groups of no more than three persons.
2. Only one submission is needed for each group. Late submission will not be accepted.
3. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
4. **Do not panic.**

Name	ID
Prapun	555

$m=2$

Consider a system which has 2 channels. We would like to find the blocking probability via the Markov chain method. For each of the following models, **<u>draw the Markov chain</u>** via discrete time approximation. Don't forget to indicate the transition probabilities on the arrows. Assume that the duration of each time slot is 1 millisecond. Then, use global balance equations to find (1) the **<u>steady-state probabilities</u>** and then (2) the long-term **<u>call blocking probability</u>**.

$\delta = 10^{-3}$

$\lambda = 12$ calls/hour

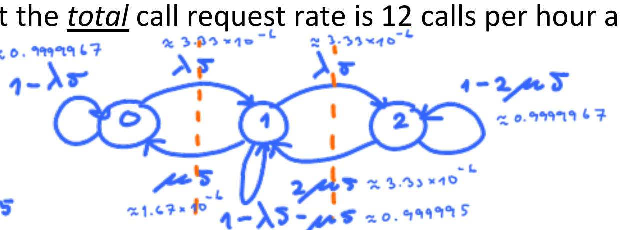
1. ***Erlang B*** model: Assume that the *total* call request rate is 12 calls per hour and the average call duration is 10 mins.

$\frac{1}{\mu} =$ 10 mins.

$\lambda\delta = \frac{\cancel{12}^{2} \times 10^{-3}}{\cancel{60} \times 60} = \frac{10^{-5}}{3} \approx 3.33\times 10^{-6}$

$\mu\delta = \frac{1}{10 \text{ min}} \times 10^{-3} \text{ sec} = \frac{10^{-3}}{10\times 60} = \frac{10^{-5}}{6} \approx 1.67\times 10^{-6}$

$A = \frac{\lambda}{\mu} = \frac{\cancel{12}^{2}}{\cancel{60}} \times 1\cancel{0} = 2$ Erlangs



$\lambda\cancel{\delta} p_0 = \mu\cancel{\delta} p_1$

$p_1 = \frac{\lambda}{\mu} p_0 = A p_0$

$\lambda\cancel{\delta} p_1 = 2\mu\cancel{\delta} p_2$

$p_2 = \frac{1}{2}\frac{\lambda}{\mu} p_1 = \frac{1}{2} A p_1 = \frac{1}{2} A^2 p_0$

$p_0 + p_1 + p_2 = 1$

$p_0 + A p_0 + \frac{1}{2} A^2 p_0 = 1$

$p_0 = \frac{1}{1 + A + \frac{1}{2}A^2} = \boxed{\frac{1}{5}} \approx 0.2$

$p_1 = \boxed{\frac{2}{5}} \approx 0.4$

$p_2 = \frac{1}{\cancel{2}} \times 2^{\cancel{2}} \times \frac{1}{5} = \boxed{\frac{2}{5}} \approx 0.4$

$P_b = P_m = P_2 = \boxed{\frac{2}{5}} \approx 0.4$

2. ***Engset*** model: Assume that there are 6 users. The call request rate for *each* user is 2 calls per hour and the average call duration is 10 mins. $= \frac{1}{\mu}$

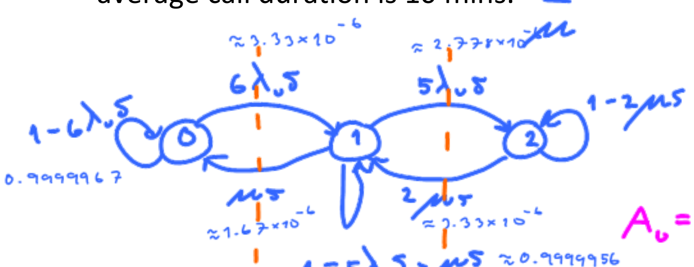
$n =$ 6

$\lambda_u =$

$\lambda_u \delta = \frac{2}{(60)^2} \times 10^{-3} = \frac{1}{18} \times 10^{-5}$

$\mu\delta = \frac{10^{-5}}{6}$

$A_u = \frac{\lambda_u}{\mu} = \frac{2}{60} \times 10 = \frac{1}{3}$



Global balance equation

$6\lambda_u\cancel{\delta} p_0 = \mu\cancel{\delta} p_1$

$p_1 = 6A_u p_0$

$5\lambda_u\cancel{\delta} p_1 = 2\mu\cancel{\delta} p_2$

$p_2 = \frac{5}{2} A_u p_1 = 15 A_u^2 p_0$

$p_0 + p_1 + p_2 = 1$

$p_0 + 6A_u p_0 + 15A_u^2 p_0 = 1$

$p_0 = \frac{1}{1 + 6A_u + 15A_u^2} = \frac{1}{1 + 6\cdot\frac{1}{3} + \cancel{15}^{5} \times \frac{1}{\cancel{9}_{3}}} = \frac{14}{3} = \boxed{\frac{3}{14}} \approx 0.214$

$p_1 = 6\cdot\frac{1}{3}\cdot\frac{3}{14} = \boxed{\frac{3}{7}} \approx 0.429$

$p_2 = \cancel{15}^{5} \times \frac{1}{\cancel{9}} \times \frac{\cancel{3}}{14} = \boxed{\frac{5}{14}} \approx 0.357$

$P_b = \frac{4\cancel{\lambda_u\delta} p_2}{6\cancel{\lambda_u\delta} p_0 + 5\cancel{\lambda_u\delta} p_1 + 4\cancel{\lambda_u\delta} p_2} = \frac{4\cdot\frac{5}{14}}{6\cdot\frac{3}{14} + 5\cdot\frac{6}{14} + 4\cdot\frac{5}{14}} = \frac{20}{18+30+20} = \frac{20}{68} = \boxed{\frac{5}{17}} \approx 0.294$

ECS 455: Quiz 4 Solution

Instructions

1. Separate into groups of no more than three persons.
2. Only one submission is needed for each group. Late submission will not be accepted.
3. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
4. **Do not panic.**

Name	ID
Prapun	555

Consider a system which has 2 channels. We would like to find the blocking probability via the Markov chain method. For each of the following models, **draw the Markov chain** via discrete time approximation. Don't forget to indicate the transition probabilities on the arrows. Assume that the duration of each time slot is 1 millisecond. Then, use global balance equations to find (1) the **steady-state probabilities** and then (2) the long-term **call blocking probability**.

$\delta = 10^{-3}$ sec

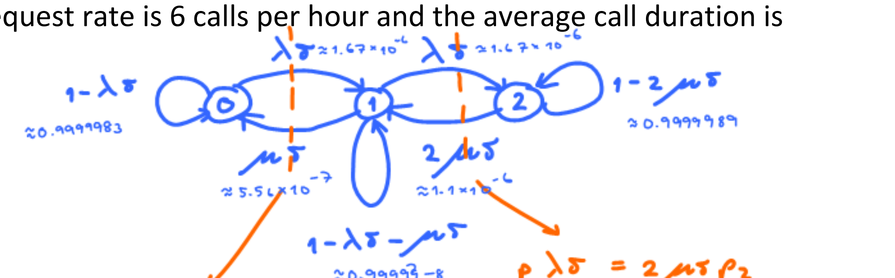
1. ***Erlang B*** model: Assume that the *total* call request rate is 6 calls per hour and the average call duration is 30 mins.

$\lambda = 6 \frac{\text{calls}}{\text{hr}} = \frac{6}{60} \frac{\text{calls}}{\text{min}} = \frac{6}{3600} \frac{\text{calls}}{\text{sec}}$

$\lambda\delta = \frac{6}{3600} \times 10^{-3} = \frac{1}{6} \times 10^{-5} \approx 1.67\times10^{-6}$

$\mu\delta = \frac{1}{30\times60} \times 10^{-3} = \frac{1}{18} \times 10^{-5} \approx 5.56\times10^{-7}$

$A = \frac{\lambda}{\mu} = \lambda \times \frac{1}{\mu} = \frac{6}{60} \times 30 = 3$ Erlangs



$p_0 \lambda\delta = p_1 \mu\delta$

$p_1 = A p_0$

$p_1 \lambda\delta = 2\mu\delta p_2$

$p_2 = \frac{A}{2} p_1 = \frac{A^2}{2} p_0$

$p_0+p_1+p_2 = 0 \Rightarrow p_0 = \frac{1}{1+A+\frac{A^2}{2}} = \frac{1}{1+3+\frac{3^2}{2}} = \frac{2}{17}$ (≈0.118) $\Rightarrow p_1 = 3\times\frac{2}{17} = \frac{6}{17}$ (≈0.353), $p_b = p_2 = \frac{3}{2}\times\frac{6}{17} = \frac{9}{17}$ (≈0.529).

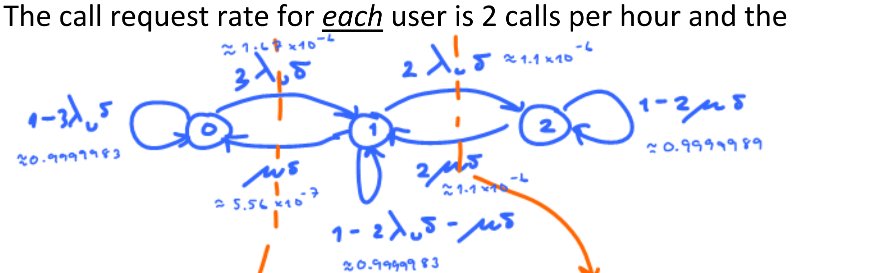
2. ***Engset*** model: Assume that there are 3 users. The call request rate for *each* user is 2 calls per hour and the average call duration is 30 mins.

$\lambda_u = 2 \frac{\text{calls}}{\text{hr}}$ ← observe that this is $\frac{\lambda}{3}$

$\lambda_u\delta = \frac{\lambda}{3}\delta = \frac{1}{18} \times 10^{-5} \approx 5.56\times10^{-7}$

$\mu\delta = \frac{1}{18}\times10^{-5} \approx 5.56\times10^{-7}$ ← same as in Q1.

$A_u = \frac{\lambda_u}{\mu} = \frac{\lambda}{3\mu} = \frac{A}{3} = \frac{3}{3} = 1$ Erlang.



$p_0 3\lambda_u\delta = p_1 \mu\delta$

$p_1 = 3A_u p_0$

$p_1 2\lambda_u\delta = p_2 2\mu\delta$

$p_2 = A_u p_1 = 3A_u^2 p_0$

$p_0+p_1+p_2 = 1 \Rightarrow p_0 = \frac{1}{1+3A_u+3A_u^2} = \frac{1}{7} \approx 0.143$

$p_1 = 3A_u p_0 = 3p_0 = \frac{3}{7} \approx 0.429$

$p_2 = 3A_u^2 p_0 = 3p_0 = \frac{3}{7} \approx 0.429$

$p_b = \frac{\frac{3}{7}\times\lambda_u\delta}{\frac{1}{7}\times3\lambda_u\delta + \frac{3}{7}\times2\lambda_u\delta + \frac{3}{7}\times\lambda_u\delta} = \frac{3}{3+6+3} = \frac{1}{4} \approx 0.25$

ECS 455: Quiz 5 **Solution**

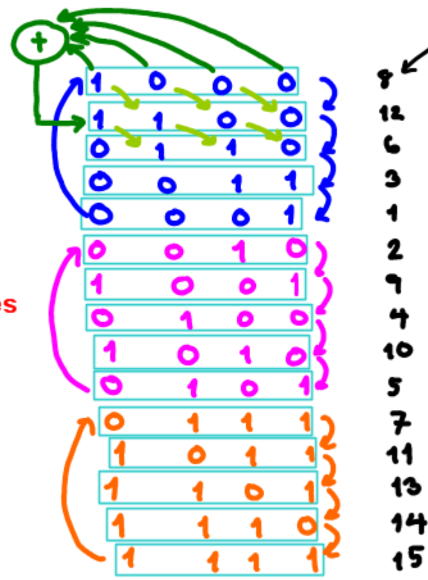
Instructions

1. Separate into groups of no more than three persons.
2. Only one submission is needed for each group. Late submission will not be accepted.
3. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
4. **Do not panic.**

Name	ID
Prapun	

Draw the complete state diagrams for linear feedback shift registers (LFSRs) using the following polynomials. Does either LFSR generate an m-sequence?

1. $x^4 + x^3 + x^2 + x + 1$



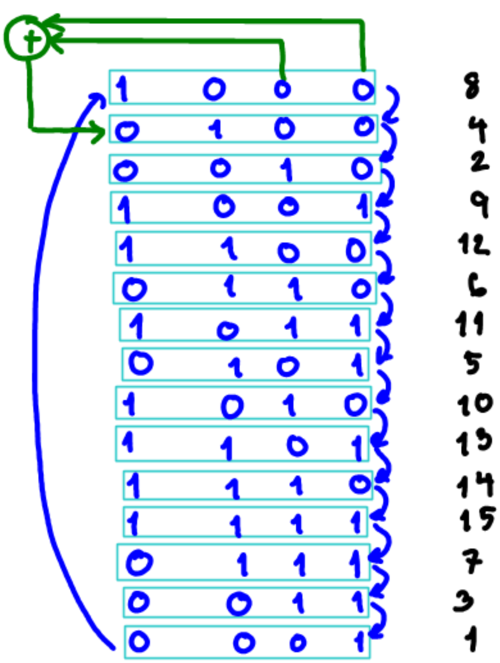
I convert the states to decimal numbers so that it is easy to see which states haven't appeared.

The corresponding LFSR does not generate m sequence because no single cycle visit all non-zero states.

Note that because the question days "complete" state diagrams, we need to show these two cycles as well.

2. $x^4 + x^3 + 1$

$1 + x^3 + x^4$



The corresponding LFSR generates an m sequence because one single cycle visit all non-zero states.

ECS 455: Quiz 6 **Solution**

Instructions

1. Separate into groups of no more than three persons.
2. Only one submission is needed for each group. Late submission will not be accepted.
3. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
4. **Do not panic.**

Name	ID

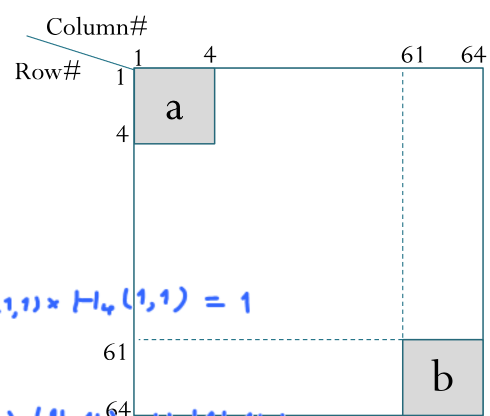
A 64×64 Hadamard matrix is created in `MATLAB` via the command

`H = hadamard(64).`

Note that the elements of `H` are all 1 or -1. Of course, there are 4,096 elements in `H`. Writing them all down would take too much time. So, in this question, you are asked to identify only parts a and b that are shown in the following picture:

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H_4 = H_2 \otimes H_2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$



$$H_{64} = H_{16} \otimes H_4$$

$$H_{16} = H_4 \otimes H_4$$

$$H_{16}(1,1) = H_4(1,1) \times H_4(1,1) = 1$$

$$H_{16}(16,16) = H_4(16,16) \times H_4(16,16) = 1$$

Remark: The picture is not drawn to scale.

a. (4 pt) Find `H(1:4,1:4)`. (This is the part of H that is denoted by (a) in the picture above. It covers rows 1 to 4 and columns 1 to 4.)

$$[a] = \underbrace{H_{16}(1,1)}_{1} \times H_4 = H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

b. (2 pt) Find `H(61:64,61:64)`.

$$[b] = \underbrace{H_{16}(16,16)}_{1} \times H_4 = H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$